Change of Base of a Number

Although base 10 (decimal[1]) is the most common base for writing numbers and base 2 (binary[2]) is widely used in computing, numbers may be written in any base. In this document we explore a method for converting a number in one base into its representation in another base.

For example consider having to convert 615423.1206 (in base 7) to base 3.

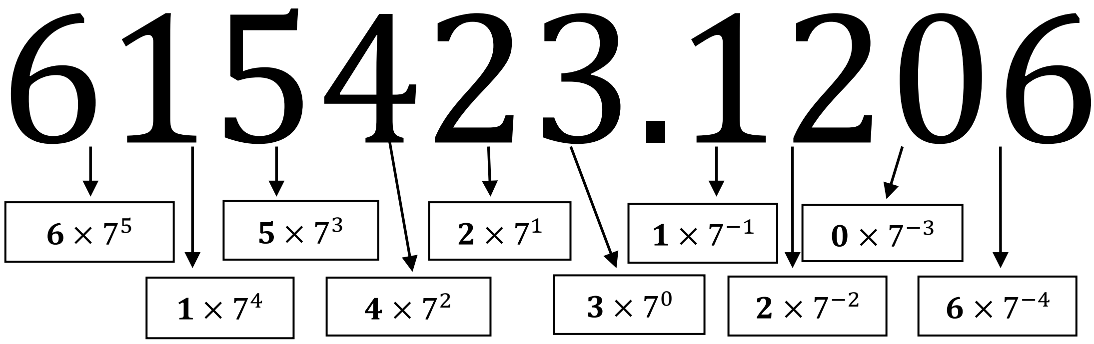
Often the base is written as a subscript to the number. For the example,

615423.1206_7 = 615423.1206 (in base 7).

Step 1. Convert to base 10.

The first step in the method is to write the number into base 10.

It is best to illustrate the method by an example. For example assume that the number that we wish to convert is



Hence $615423.1206_7 = 6\times7^5 + 1\times7^4 + 5\times7^3 + 4\times7^2 + 2\times7^1 + 3\times7^0 + 1\times7^{-1} + 2\times7^{-2} + 0\times7^{-3} + 6\times7^{-4} = 6\times16807 + 1\times2401 + 5\times343 + 4\times49 + 2\times7 + 3\times1 + 1\times0.142857 + 2\times0.020408 + 0\times0.002915 + 6\times0.000416$ =105171.1862

That is

$$615423.1206_7 = 105171.1862_{10}$$

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[1] Decimal Representation of Numbers

[2] Binary Representation of Numbers

Step 2. Convert to the new base

If we carry out repeated division of the whole part of the number in base 10 by the new base and read back through the remainders we get the whole part of the number in the new base.

For example for 105171_{10}:

$$105171 \div 3 = 35057 \text{ rem } 0$$

$$35057 \div 3 = 11685 \text{ rem } 2$$

$$11685 \div 3 = 3895 \text{ rem } 0$$

$$3895 \div 3 = 1298 \text{ rem } 1$$

$$1298 \div 3 = 432 \text{ rem } 2$$

$$432 \div 3 = 114 \text{ rem } 0$$

$$114 \div 3 = 48 \text{ rem } 0$$

$$48 \div 3 = 16 \text{ rem } 0$$

$$16 \div 3 = 5 \text{ rem } 1$$

$$5 \div 3 = 1 \text{ rem } 2$$

$$1 \div 3 = 0 \text{ rem } 1$$

The whole number part of the number in base 3 is achieved by reading backwards through the remainders; 12100021020_3.

For the remaining part of the number, we repeatedly multiply by the new base, removing the whole part of the number at each stage.

$$0.1862 \times 3 = 0.5586 = 0 + 0.5586$$

$$0.5586 \times 3 = 1.6758 = 1 + 0.6758$$

$$0.6756 \times 3 = 2.0274 = 2 + 0.0274$$

$$0.0274 \times 3 = 0.0822 = 0 + 0.0822$$

$$0.0822 \times 3 = 0.2466 = 0 + 0.2466$$

$$0.2466 \times 3 = 0.7398 = 0 + 0.7398$$

$$0.7398 \times 3 = 2.2194 = 2 + 0.2194$$

This method can continue, depending on the accuracy required.
The fractional part of the number in the new base is achieved by reading through the integer parts: $.0120002_3$.

Hence the full result is $615423.1206_7 = 12100021020.0120002_3$.

Spreadsheet

The accompanying spreadsheet[3] implements the method outlined. The number of digits before and after is pre-set. It is possible that the result will not fit in the blue row, in this case an "ERROR" is flagged.

original number, original base and																								www.mathematics.kirkup.info	
new base in the yellow areas																number									base
								5	6	2	1	0	4	1	3	•	1	2	0	6	0	0	0	0	7
0	1	0	0	0	1	0	2	2	0	1	0	2	2	0	1	•	0	1	2	0	0	0	2	0	3

3 Change of Base – (Spreadsheet)